RADIATION OF ELASTIC WAVES IN ONE-DIMENSIONAL SYSTEMS BY A MOVING SOURCE

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Radiation of waves in a medium by a uniformly moving source of given frequency Ω is characterized by the Doppler effect [1], the shift of the frequency of the waves being radiated as compared with the frequency of the source. In the case $\Omega = 0$, the radiation phenomenon takes the form of the Vavilov–Cerenkov effect [2] and is possible only under the condition that the velocity of source motion exceeds the least phase velocity of wave propagation in the medium [3].

The phenomena mentioned are studied in sufficient detail in application to electrodynamic systems [4, 5]. For elastic systems, where a moving local field of deformation caused, say, by the effect of a mobile load can be the source, such phenomena have singularities, even in the one-dimensional case, due to the specifics of elastic wave dispersion. The study of these singularities acquires special importance because of the need to solve questions of vibration excitation in application to a broad circle of technical systems and apparatus: power transmissions with flexible couplings, tape drive and rewind mechanisms [6], rope lifts, the rolling and extrusion of wires [7], etc.

Regimes of wave excitation by a moving vibrating fastening are examined in this paper in an example on the bending vibrations of a rod.

Let us consider a homogeneous infinite rod along which a bushing moves at a constant velocity v to perform a turning motion with frequency Ω relative to the center of gravity. The bending vibrations of a rod $U(x, t)$ in the approximation of engineering theory [8] are described by the equation

$$U_{tt} + \alpha^2 U_{xxxx} = 0 \tag{1}$$

with the conditions on the moving support

$$U|_{x=-vt} = 0, \ U_x|_{x=-vt} = \theta \cos \Omega t, \tag{2}$$

where $\alpha^2 = \beta/\rho$; β is the bending stiffness coefficient; ρ, linear density of the rod; and θ, amplitude of bushing vibrations.

Assuming the process of bending vibration excitation steady, we seek the solution of the problem (1), (2) to the left (Domain I) and right (Domain II, see Fig. 1) of the boundary $x = -vt$ in the form of the traveling waves

$${}^{\mathrm{I,II}}U(x, t) = {}^{\mathrm{I,II}}A \cos ({}^{\mathrm{I,II}}\omega t + {}^{\mathrm{I,II}}kx + {}^{\mathrm{I,II}}\varphi), \tag{3}$$

where among the possible solutions (3) we shall consider those physically realizable which correspond to limited deflections at infinity and satisfy the Mandelshtam radiation condition [9], i.e., remove energy from the source.

The frequencies and wave numbers of the radiated waves are determined from the equation

$$-\omega^2 + \alpha^2 k^4 = 0, \ \omega - vk = \Omega, \tag{4}$$

where the first determines the dispersion properties of the system, while the second expresses equality of the phase of the waves (3) to the phase of the source vibrations for $x = -vt$.

Let us first examine the case of radiation by a constant source, i.e., by a source whose frequency equals zero ($\Omega = 0$). Determining the wave numbers and frequencies from (4), as well as the amplitudes of the vibrations being excited, we obtain the following solution from (2):

$${}^{\mathrm{I}}U(x, t) = (\theta\alpha/v) \cos (v^2 t/\alpha + vx/\alpha + \pi/2), \ {}^{\mathrm{II}}U(x, t) = 0.$$

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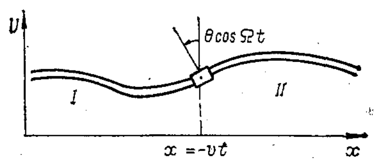

Fig. 1

It is a wave removing energy at the rate $v_b = 2v$, $v_b = d\omega/dk$ in the direction of boundary motion $-x$. The appearance of such wave is caused, due to the Vavilov–Cerenkov effect, by radiation of the energy of a constant moving deformation field due to a static bending moment on the boundary $x = -vt$. The singularity in the appearance of this effect is expressed by the fact that the radiation sets in for an arbitrarily low velocity of source motion since the least phase velocity of bending wave propagation for this model of a rod is zero.

In the case of a periodic source $\Omega \neq 0$, by solving (4) we select just those pairs of values of ω and k to which physically realizable solutions (3) will correspond:

$$
\begin{aligned}
{}^{I}k_{1,2} &= \frac{\pm v \pm \sqrt{v^2 \pm 4\alpha\Omega}}{2\alpha}, \quad {}^{I}\omega_{1,2} = \frac{\pm v^2 \pm v\sqrt{v^2 \pm 4\alpha\Omega} + 2\alpha\Omega}{2\alpha},\\
{}^{II}k_{1,2} &= \frac{\pm v \mp \sqrt{v^2 \pm 4\alpha\Omega}}{2\alpha}, \quad {}^{II}\omega_{1,2} = \frac{\pm v^2 \mp v\sqrt{v^2 \pm 4\alpha\Omega} + 2\alpha\Omega}{2\alpha}.
\end{aligned}
\tag{5}
$$

It follows from (5) that the vibration radiation process is characterized by two qualitatively distinct regimes depending on the value of v.

For velocities $v < 2\sqrt{\alpha\Omega}$ the solution (3) has the form

$$
\begin{aligned}
{}^{I}U(x,t) = {}^{I}U_1 + {}^{I}U_2 = \frac{-\theta\alpha}{\left(v^2 + v\sqrt{v^2+4\alpha\Omega} + 2\alpha\Omega\right)^{1/2}} \Bigg[\cos\left(\frac{v^2 + v\sqrt{v^2+4\alpha\Omega}+2\alpha\Omega}{2\alpha}t + \frac{v+\sqrt{v^2+4\alpha\Omega}}{2\alpha}x + {}^{I}\varphi\right) - \\
- \exp\left[\frac{(4\alpha\Omega - v^2)^{1/2}}{2\alpha}(x+vt)\right]\cos\left(\frac{2\alpha\Omega - v^2}{2\alpha}t - \frac{v}{2\alpha}x + {}^{I}\varphi\right)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
{}^{II}U(x,t) = {}^{II}U_1 + {}^{II}U_2 = \frac{-\theta\alpha}{\left(v^2 - v\sqrt{v^2+4\alpha\Omega} + 2\alpha\Omega\right)^{1/2}} \Bigg[\cos\left(\frac{v^2 - v\sqrt{v^2+4\alpha\Omega}+2\alpha\Omega}{2\alpha}t + \frac{v+\sqrt{v^2+4\alpha\Omega}}{2\alpha}x + {}^{II}\varphi\right) - \\
- \exp\left[\frac{(4\alpha\Omega - v^2)^{1/2}}{-2\alpha}(x+vt)\right]\cos\left(\frac{2\alpha\Omega - v^2}{2\alpha}t - \frac{v}{2\alpha}x + {}^{II}\varphi\right)\Bigg],
\end{aligned}
$$

where ${}^{I}\varphi = \operatorname{arctg}\left[\frac{2v + (v^2+4\alpha\Omega)^{1/2}}{(4\alpha\Omega - v^2)^{1/2}}\right]$; ${}^{II}\varphi = \operatorname{arctg}\left[\frac{2v - (v^2+4\alpha\Omega)^{1/2}}{(4\alpha\Omega - v^2)^{1/2}}\right]$, and is four waves: two to the left and two to the right of the source ${}^{I}U_{1,2}$ and ${}^{II}U_{1,2}$. Two have constant amplitude and remove energy from the source at the rates

$$ {}^{I,II}v_{b1} = v \pm \sqrt{v^2 + 4\alpha\Omega} $$

in the directions $\mp x$, respectively. The other two waves do not take part in the energy transfer since their group velocities agree in modulus and direction with v while the amplitudes tend to zero as $x \to \mp\infty$. These inhomogeneous waves are local characteristics of the source for radiation.

This regime is qualitatively not different from the radiation regime of a fixed source $v = 0$. According to the Doppler effect, the motion just results in a change in the spatial characteristics and the frequencies of the radiated waves. By calculating the energy density averaged with respect to the period $\varepsilon = \frac{1}{2}(\rho U_t^2 + \beta U_{xx}^2)$ and the energy flux density $S = \varepsilon v_b$ of these waves, it can be shown that the radiation process is characterized by the following adiabatic invariants

$$ \frac{{}^{I}\varepsilon_1}{{}^{I}\omega_1} = \frac{{}^{II}\varepsilon_1}{{}^{II}\omega_1} = \frac{1}{4}\theta^2(\beta\rho)^{1/2}, \quad \frac{{}^{I}S_1}{{}^{I}\omega_1{}^{I}k_1} = \frac{{}^{II}S_1}{{}^{II}\omega_1{}^{II}k_1} = \frac{1}{2}\theta^2\beta. $$

Let us note that analogous invariants are also satisfied in the case of a double Doppler effect [10, 11].

For $v > 2\sqrt{\alpha\Omega}$ the solution (3) is also the superposition of four waves from the source to the left and right, respectively:

$$ {}^{I}U(x,t) = {}^{I}U_1 + {}^{I}U_2 = \frac{-2\alpha\theta}{2v + (v^2+4\alpha\Omega)^{1/2} + (v^2 - 4\alpha\Omega)^{1/2}} \times $$

$$\times\left[\cos\left(\frac{v^2+v\sqrt{v^2+4\alpha\Omega}+2\alpha\Omega}{2\alpha}t+\frac{v+\sqrt{v^2+4\alpha\Omega}}{2\alpha}x+\frac{\pi}{2}\right)+\right.$$

$$\left.+\cos\left(\frac{v^2+v\sqrt{v^2-4\alpha\Omega}+2\alpha\Omega}{2\alpha}t+\frac{v+\sqrt{v^2-4\alpha\Omega}}{2\alpha}x-\frac{\pi}{2}\right)\right],$$

$${}^{II}U(x,t)={}^{II}U_1+{}^{II}U_2=\frac{-2\alpha\theta}{2v-(v^2+4\alpha\Omega)^{1/2}-(v^2-4\alpha\Omega)^{1/2}}\times$$

$$\times\left[\cos\left(\frac{v^2-v\sqrt{v^2+4\alpha\Omega}+2\alpha\Omega}{2\alpha}t+\frac{v-\sqrt{v^2+4\alpha\Omega}}{2\alpha}x+\frac{\pi}{2}\right)+\right.$$

$$\left.+\cos\left(\frac{v^2-v\sqrt{v^2-4\alpha\Omega}+2\alpha\Omega}{2\alpha}t+\frac{v-\sqrt{v^2-4\alpha\Omega}}{2\alpha}x-\frac{\pi}{2}\right)\right].$$

Each of the waves has a constant amplitude and takes part in energy transfer. The waves ${}^{I}U_{1,2}$ remove energy in the direction $-x$ with the velocities

$${}^{I}v_{b1,2}=v+\sqrt{v^2\pm4\alpha\Omega},$$

while the other two waves remove energy in the opposite direction from the source at the velocity

$${}^{II}v_{b1,2}=v-\sqrt{v^2\pm4\alpha\Omega}.$$

In this case the radiation singularity is that in addition to the waves ${}^{I,II}U_1$ two other waves ${}^{I,II}U_2$ are still radiated which are long the local characteristic of the source for low velocities $v < 2\sqrt{\alpha\Omega}$. This is because radiation of "additional" energy (because of the source motion) becomes possible starting with source velocities exceeding a certain value $v = 2\sqrt{\alpha\Omega}$, by means of traveling waves of the same kind as occurs in the case of the Vavilov–Cerenkov effect.

Let us note that the results presented remain valid even for a fixed source but a moving medium. This permits observation of the mentioned radiation regimes in simple technical systems, for instance in power transmissions with flexible coupling.

Thus, Vavilov–Cerenko radiation was observed when drawing a circular rubber rod through a fixed support, for which a fluoroplastic washer was used which had an inner diameter of the same diameter as the rod d. The slope needed with respect to such a boundary was assured both by the deflection of the rod under the effect of gravity and under the effect of centrifugal forces originating during the drawing. The rod used in the experiment had the following parameters: l = 96 cm, ρ = 0.45 g/cm, d = 0.8 cm, T = 1–1.5 · 10^6 dyne, β = 381478.88 dyne · cm^2, where l is the length of the working portion of the rod, and T is the longidudinal tensile force of the rod. The rate of drawing varied between 0 and 19 m/sec.

Starting with the velocities $v > c_0$, where $c_0 = \sqrt{T/\rho}$ is the least phase velocity of wave propagation, the excitation of transverse vibrations whose spatial period diminished as v grew, was observed in the rod. Thus for T = 1,454,047.6 dynes and v = 1817 cm/sec, we have λ = 21.3 cm (Fig. 2), which with not more than 5% er-

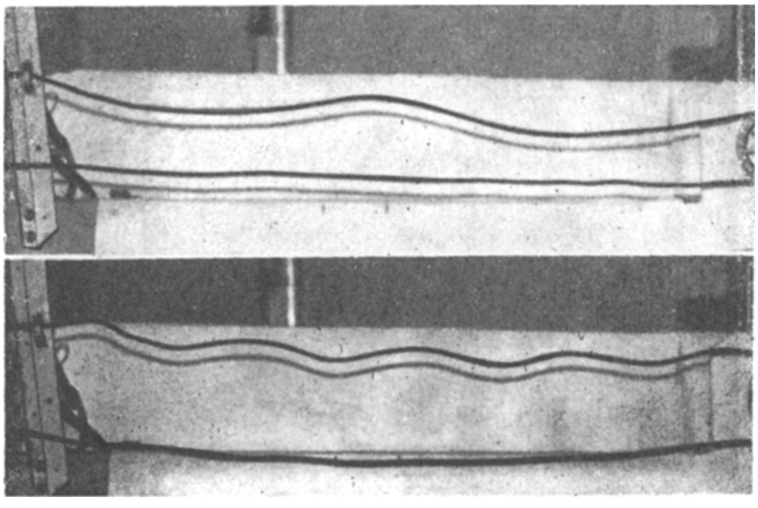

Fig. 2

ror, differs from the theoretical results. If the spatial period of the waves were multiplied by twice the length of the working section $2l = \lambda N$ ($N = 1, 2, 3, \ldots$), then the vibration amplitude would be greatest because of the resonance properties of the system.

In conclusion, let us note that since a definite frequency of the vibrations being excited corresponds to each value of v in the regimes mentioned, then excitation of vibrations with a broad spectrum is apparently possible for nonuniform source motion similarly to how the double Doppler effect results in pulse generation [12] under periodic boundary motion.

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